Confidence intervals

Announcements

- HW 4 released, due next Friday <- Note for Ciaran: modify this slightly
- Exam 1 released next Friday
  - Take home, 1 week to complete
  - Open note (anything from this course)
  - Closed internet
  - Closed other people
- Reminder: department seminar on Monday, 12pm - 1pm (Dr. Mine Cetinkaya-Rundel)
  - Can sign up to meet with the speaker 11 - 11:30

Wald vs. likelihood ratio tests

- Both can test one or more parameters
- Convention: use Wald for single parameters, LRT for multiple

typically $Z = \dfrac{\hat{\beta}_i - \beta_i^0}{\sqrt{Var(\hat{\beta}_i)}} \sim N(0,1)$

instead of $W = Z^2 \sim \chi^2_1$

- Wald tests do not require fitting a reduced model
- Wald tests are good for linear combinations of βs
- LRT does not require inverting (potentially large) matrices
- In logistic regression, LRT performs better when $\hat{p}_i \approx 0$ or 1 for some observations

95% CI : 1.192 ± 1.96 (0.243) = (0.716, 1.668)

97.5 quantile of N(0,1)

Confidence intervals

$$\log\left(\frac{p_i}{1-p_i}\right) = \beta_0 + \beta_1 Sex_i + \beta_2 Age_i + \beta_3 SecondClass_i +$$

$$\beta_4 FirstClass_i + \beta_5 Sex_i \cdot Age_i$$

vcov(model)

diagonal entries of $\mathcal{I}^{-1}(\hat{\beta})$

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...
## Coefficients:
##               Estimate Std. Error z value Pr(>|z|)
## (Intercept)   0.408232   0.330916   1.234 0.217337
## Sexmale      -1.163444   0.437622  -2.659 0.007848 **
## Age          -0.007186   0.011684  -0.615 0.538522
## Pclass2       1.191858   0.243233   4.900 9.58e-07 ***
## Pclass1       2.697561   0.295822   9.119  < 2e-16 ***
## Sexmale:Age  -0.049851   0.014782  -3.373 0.000745 ***
...
```

How do I create a 95% confidence interval for β_3?

This statement is <u>not</u> true:

$$P(\beta_3 \in (0.716, 1.668)) = 0.95$$

This statement <u>is</u> true:

$$P(\beta_3 \in (\hat{\beta}_3 - 1.96 SE(\hat{\beta}_3), \hat{\beta}_3 + 1.96 SE(\hat{\beta}_3))) = 0.95$$

Wald confidence intervals

<u>Goal</u>: want a "reasonable range" for unknown β_i

<u>Method</u>: $\hat{\beta} \approx N(\beta, \mathcal{I}^{-1}(\beta)) \Rightarrow \hat{\beta}_i \approx N(\beta_i, \underbrace{\mathcal{I}^{-1}(\beta)_{ii}}_{i^{th} \text{ diagonal entry } \mathcal{I}^{-1}(\beta)})$

<u>$1-\alpha$ confidence interval</u>: $\hat{\beta}_i \pm \underbrace{z_{1-\frac{\alpha}{2}}}_{1-\frac{\alpha}{2} \text{ quantile of } N(0,1)} \underbrace{\sqrt{\mathcal{I}^{-1}(\hat{\beta})_{ii}}}_{SE(\hat{\beta}_i)}$

<u>Ex</u>: 95% CI $\Rightarrow \alpha = 0.05 \Rightarrow 1-\frac{\alpha}{2} = 0.975$

$z_{1-\frac{\alpha}{2}} = 1.96 \Rightarrow \hat{\beta}_i \pm 1.96\sqrt{\mathcal{I}^{-1}(\hat{\beta})_{ii}}$

CIs vs. Hypothesis tests

Let $\theta \in \mathbb{R}$ be a parameter of interest, and suppose $\hat{\theta}$ is an estimate, where $\hat{\theta} \,\dot{\sim}\, N(\theta, Var(\hat{\theta}))$

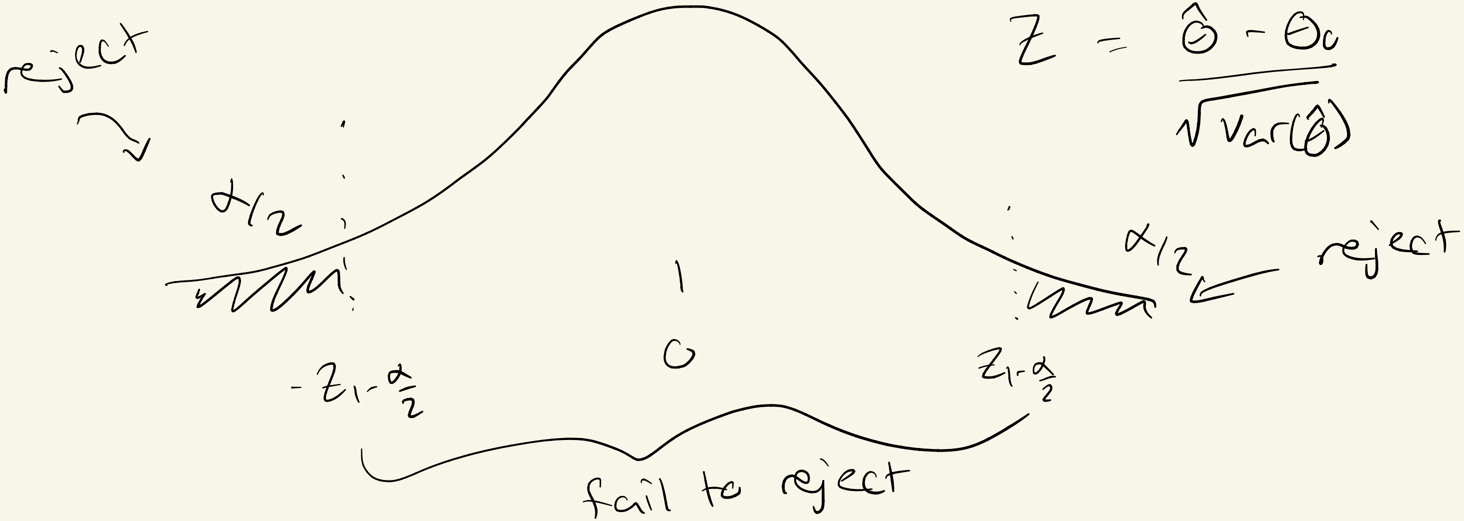
Consider testing $H_0: \theta = \theta_0$ vs. $H_A: \theta \neq \theta_0$

<u>wald test statistic</u>: $W = \dfrac{(\hat{\theta} - \theta_0)^2}{Var(\hat{\theta})} \sim \chi^2_1$ $\qquad Z = \dfrac{\hat{\theta} - \theta_0}{\sqrt{Var(\hat{\theta})}} \sim N(0,1)$

1) want a "reasonable range" of values for θ
2) Rejecting $H_0 \Rightarrow \theta = \theta_0$ is "unreasonable"
3) so, $CI = \{ \theta_0 : \text{fail to reject } H_0: \theta = \theta_0 \}$

Suppose we want type I error $= \alpha$, what is the rejection region?

$$Z < -z_{1-\frac{\alpha}{2}} \quad \text{or} \quad Z > z_{1-\frac{\alpha}{2}}$$



$$Z = \frac{\hat{\theta} - \theta_0}{\sqrt{Var(\hat{\theta})}}$$

fail to reject when

$$-Z_{1-\frac{\alpha}{2}} < \frac{\hat{\theta} - \theta_0}{\sqrt{Var(\hat{\theta})}} < Z_{1-\frac{\alpha}{2}}$$

$$\Rightarrow \hat{\theta} - Z_{1-\frac{\alpha}{2}}\sqrt{Var(\hat{\theta})} < \theta_0 < \hat{\theta} + Z_{1-\frac{\alpha}{2}}\sqrt{Var(\hat{\theta})}$$

$$\Rightarrow CI = \left[\hat{\theta} - Z_{1-\frac{\alpha}{2}}\sqrt{Var(\hat{\theta})},\ \hat{\theta} + Z_{1-\frac{\alpha}{2}}\sqrt{Var(\hat{\theta})}\right]$$

Technique: Inverting the test

Confidence intervals for linear combinations

Class activity

https://sta712-f22.github.io/class_activities/ca_lecture_15.html